Signal Analysis and Processing in Biomedicine

Prof. dr. Srdjan Stankovic
Prof. dr. Irena Orovic
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Overview

- Mathematical transforms in biomedical signal processing
- Computer tomography
- Compressive sensing
- Compressive sensing in biomedical imaging
- ECG signals
- Hermite transform in ECG signals analysis
- Detection of swallowing sounds – appl. Dysphagia
- Telemedicine
Mathematical transforms in biomedical signal processing
Fourier transform

\[ F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} \, dt \]

inverse: \[ f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} \, d\omega \]

- Some useful Properties:
  - **Linearity**
    \[ \int_{-\infty}^{\infty} (\alpha f(t)+\beta g(t))e^{-j\omega t} \, dt = \alpha \int_{-\infty}^{\infty} f(t)e^{-j\omega t} \, dt + \beta \int_{-\infty}^{\infty} g(t)e^{-j\omega t} \, dt = \alpha F(\omega)+\beta G(\omega) \]
  - **Time shift**
    \[ \int_{-\infty}^{\infty} f(t-t_0)e^{-j\omega t} \, dt = e^{-j\omega t_0} F(\omega) \]
  - **Frequency shift**
    \[ \int_{-\infty}^{\infty} \left(e^{j\omega_0 t} f(t)\right)e^{-j\omega t} \, dt = F(\omega-\omega_0) \]
  - **Convolution**
    \[ \text{FT}\{ f(t) \ast g(t)\} = \text{FT}\left\{ \int_{-\infty}^{\infty} f(\tau) g(t-\tau) \, d\tau \right\} = F(\omega)G(\omega) \]

\[ \text{FT}\{ f(t) \cdot g(t)\} = F(\omega) \ast G(\omega) \]
Discrete Fourier transform

\[ DFT(k) = \sum_{n=0}^{N-1} f(n) e^{-j \frac{2\pi}{N} nk} \]

\[ f(n) = \frac{1}{N} \sum_{k=0}^{N-1} DFT(k) e^{j \frac{2\pi}{N} nk} \]

Noisy signal

Fourier transform

$X(t) = A e^{j\phi(t)}$ - Frequency modulated signal

Ideal time-frequency representation of $x(t)$ should concentrate energy along the instantaneous frequency of the signal. It is defined as:

$$ITF(t, \omega) = 2\pi A^2 \delta(\omega - \phi'(t)),$$

where $\omega = \phi(t)$ is instantaneous frequency.
Time-Frequency Representation

Short Time-Fourier Transform

$$STFT(t, \omega) = \int_{-\infty}^{\infty} w(\tau)x(t+\tau)e^{-j\omega \tau} \, d\tau$$

window function

Multicomponent signals:

$$STFT(\omega, t) = \sum_{m=1}^{M} STFT_{x_m}(\omega, t)$$

$$x(t) = \sum_{m=1}^{M} x_m(t)$$

STFT is linear transform
S-method

- S-method: Distribution with the auto-terms being the same as in the Wigner distribution, but with reduced or completely removed cross-terms:

\[
SM(\omega, t) = \frac{1}{\pi} \int_{-\infty}^{\infty} P(\theta) STFT(\omega + \theta, t) STFT^*(\omega - \theta, t) d\theta,
\]

Where \( P(\theta) \) is finite frequency domain window. Discrete form of the S-method is:

\[
SM(k, n) = \sum_{i=-L_d}^{L_d} P(i) STFT(k+i, n) STFT^*(k-i, n) =
\]

\[
= |STFT(k, n)|^2 + 2 \cdot \text{Re} \left\{ \sum_{i=1}^{L_d} STFT(k+i, n) STFT^*(k-i, n) \right\}
\]

In order to avoid the presence of cross-terms, the value of \( L_d \) should be less than half of the distance between two auto-terms.
Wavelet transform

**Continuous wavelet transform:**

- Wavelets can be used as basis for representing functions:
  \[ y(t) = \sum_i a_i \psi_i(t) \]

- Wavelets-functions formed by scaling and translation of basis (mother) functions in the time domain;

- It satisfies the following conditions:

<table>
<thead>
<tr>
<th>Condition</th>
<th>Mathematical Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area under the curve is equal to zero</td>
<td>[ \int_{-\infty}^{\infty} \psi(t) dt = 0 ]</td>
</tr>
<tr>
<td>The function is square integrable</td>
<td>[ \int_{-\infty}^{\infty}</td>
</tr>
</tbody>
</table>
Wavelet transform

- Wavelet is defined by formula:
  \[ \psi_{a,b}(t) = \frac{1}{\sqrt{|a|}} \psi \left( \frac{t-b}{a} \right) \]
  - \( a, b \) - two real numbers used for scaling and translation in time
  - \( 0 < a < 1 \) basis function is shrunk in time
  - \( a > 1 \) basis function is spread

- Continuous wavelet transform of the signal is given by:
  \[ W(a,b) = \int_{-\infty}^{\infty} \psi_{a,b}(t) f(t) dt \]

- Inverse form:
  \[ f(t) = \frac{1}{C} \int_{a=-\infty}^{\infty} \int_{b=-\infty}^{\infty} \psi_{a,b}(t) W(a,b) dadb \]

- Fourier transform of \( \psi(t) \)
  \[ C = \int_{-\infty}^{\infty} \left| \Psi(\omega) \right|^2 d\omega \]
  - \( \Psi(\omega) \) is positive and finite
Wavelet families

- Daubeshies
- Haar
- Biorthogonal
- Mexican Hat
- Symlets
- Morlet
- Coiflets
- Meyer
Discrete wavelets

- Discretization is done by using:
  \[ a = a_0^m, \quad b = n b_0 a_0^m \]
  \[ m, n \in \mathbb{Z}, \quad b_0 > 0 \]

Discrete wavelet function

\[ \psi_{m,n}(t) = a_0^{-m/2} \psi(a_0^{-m} t - n b_0) \]

Discrete wavelet transformation

\[ W_{m,n}^d = a_0^{-m/2} \int_{-\infty}^{\infty} f(t) \psi(a_0^{-m} t - n b_0) dt \]

- Using \( a_0 = 2 \) and \( b_0 = 1 \) dyadic sampling is obtained

\[ \psi_{m,n}(t) = 2^{-m/2} \psi(2^{-m} t - n) \]

\[ W_{m,n}^d = 2^{-m/2} \int_{-\infty}^{\infty} f(t) \psi(2^{-m} t - n) dt \]
Standard wavelet decomposition

- Decomposition is applied to rows, then to columns
  - $L_i$ - low-frequency sub-band
  - $H_i$ - high-frequency sub-band
Quincunx decomposition

- Decomposition uses only low frequency sub-band of different level
- Low frequency sub-band is then divided into low and high frequency part
Pyramidal wavelet decomposition

Uniform wavelet decomposition
The Hermite functions provides good localization and the compact support in both time and frequency domain. The $i$-th order Hermite function is defined as:

$$
\psi_i(t) = (-1)^i \frac{e^{t^2/2}}{\sqrt{2^i i! \sqrt{\pi}}} \cdot \frac{d^i (e^{-t^2})}{dt^i}.
$$

**Recursive realization of Hermite functions:**

$$
\psi_0(x) = \frac{1}{\sqrt{4\pi}} e^{-x^2/2}, \quad \psi_1(x) = \frac{\sqrt{2}x}{\sqrt{4\pi}} e^{-x^2/2}, \quad \psi_p(x) = x \sqrt{\frac{2}{p}} \psi_{p-1}(x) - \sqrt{\frac{p-1}{p}} \psi_{p-2}(x), \ \forall p \geq 2.
$$
Hermite expansion

First step – removing the baseline

Baseline subtraction from the original signal

Decomposition using \( N \) Hermite functions

Hermite expansion coefficients:

\[
c_p(i) = \sum_{m=1}^{\infty} \mu_{M-1}^p(x_m) f(x_m)
\]

- Gauss-Hermite quadrature technique can be used to calculate the Hermite expansion coefficients

- a zeros of an \( M \)-th order Hermite polynomial
<table>
<thead>
<tr>
<th>Hermite Polynomials</th>
<th>Zeros</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_1(x) = 2x$</td>
<td>0</td>
</tr>
<tr>
<td>$H_2(x) = 4x^2 - 2$</td>
<td>±0.707</td>
</tr>
<tr>
<td>$H_3(x) = 8x^3 - 12x$</td>
<td>±1.2247, 0</td>
</tr>
<tr>
<td>$H_4(x) = 16x^4 - 48x^2 + 12$</td>
<td>±1.6507, ±0.5246</td>
</tr>
<tr>
<td>$H_5(x) = 32x^5 - 160x^3 + 120x$</td>
<td>±2.0202, ±0.9586, 0</td>
</tr>
<tr>
<td>$H_6(x) = 64x^6 - 480x^4 + 720x^2 - 120$</td>
<td>±2.3506, ±1.3358, ±0.4361</td>
</tr>
<tr>
<td>$H_7(x) = 128x^7 - 1344x^5 + 3360x^3 - 1680x$</td>
<td>±2.6520, ±1.6736, ±0.8163, 0</td>
</tr>
<tr>
<td>$H_8(x) = 256x^8 - 3584x^6 + 13440x^4 - 13440x^2 + 1680$</td>
<td>±2.9306, ±1.9817, ±1.1572, ±0.3812</td>
</tr>
<tr>
<td>$H_9(x) = 512x^9 - 9216x^7 + 48384x^5 - 80640x^3 + 30240x$</td>
<td>±3.1910, ±2.2666, ±1.4686, ±0.7236, 0</td>
</tr>
<tr>
<td>$H_{10}(x) = 1024x^{10} - 23040x^8 + 161280x^6 - 403200x^4 + 302400x^2 - 30240$</td>
<td>±3.4362, ±2.5327, ±1.7567, ±1.0366, ±0.3429</td>
</tr>
</tbody>
</table>
Hermite transform

The expansion using $N$ Hermite functions can be written in matrix form. First, we define the Hermite transform matrix $W$ (of size $N \times N$):

$$
W = \frac{1}{N} \begin{bmatrix}
\psi_0(1) & \psi_0(2) & \psi_0(M) \\
(\psi_{N-1}(1))^2 & (\psi_{N-1}(2))^2 & (\psi_{N-1}(M))^2 \\
\psi_1(1) & \psi_1(2) & \psi_1(M) \\
(\psi_{N-1}(1))^2 & (\psi_{N-1}(2))^2 & (\psi_{N-1}(M))^2 \\
\vdots & \vdots & \vdots \\
\psi_{N-1}(1) & \psi_{N-1}(2) & \psi_{N-1}(M) \\
(\psi_{N-1}(1))^2 & (\psi_{N-1}(2))^2 & (\psi_{N-1}(M))^2 
\end{bmatrix}
$$

If the vector of Hermite coefficients is: $c = [c_0, c_1, \ldots, c_{N-1}]^T$

and vector of $M$ signal samples is: $f = [f(1), f(2), \ldots, f(M)]^T$

then we have

$$
c = Wf
$$

For a signal of length $M$, the complete set of discrete Hermite functions consists of exactly $N=M$ function. In some applications, a smaller number of Hermite functions $N<M$ can be used.
Having in mind the Gauss-Hermite approximation, the inverse matrix $w^{-1}$ contains $N$ Hermite functions, given by:

$$
\Psi = \begin{bmatrix}
\psi_0(1) & \psi_0(2) & \ldots & \psi_0(M) \\
\psi_1(1) & \psi_1(2) & \ldots & \psi_1(M) \\
\vdots & \vdots & \ddots & \vdots \\
\psi_{N-1}(1) & \psi_{N-1}(2) & \ldots & \psi_{N-1}(M)
\end{bmatrix} = W^{-1}
$$

Now, the Hermite expansion for the case of discrete signal can be defined as follows:

$$
f = W^{-1}c = \Psi c
$$
Computer tomography
Clinical Computed Tomography (CT) was introduced in 1971 - limited to axial imaging of the brain in neuroradiology. It developed into a versatile 3D whole body imaging modality for a wide range of applications in for example oncology, vascular radiology, cardiology, traumatology and interventional radiology.

Computed tomography can be used for diagnosis and follow-up studies of patients, planning of radiotherapy treatment, screening of healthy subpopulations with specific risk factors.
Computer Tomography

- CT scanning is suitable for 3D imaging of the brain, cardiac, musculoskeletal, and whole body CT imaging.
- The images can be viewed as colored 3D rendered images, although radiologists prefer black and white 2D images.
Computer Tomography

- Acquisition is achieved using:
  - hundreds of detector elements along the detector arc (800-900 detector elements),
  - using rotation of the x ray tube around the patient, taking about 1000 angular measurements
  - using tens or even hundreds of detector rows aligned next to each other along the axis of rotation
Image reconstructions from projections

- Image reconstruction based on projections has important applications in various fields (e.g., *in medicine when dealing with computer tomography, which is widely used in everyday diagnosis*).

- Consider an object in space, which can be described by the function \( f(x,y) \). The projection of function \( f(x,y) \) along an arbitrary direction (defined by an angle \( \phi \)) can be defined as follows:

\[
p_\phi(u) = \int_{AB} f(x,y) dl, \quad u = x \cos \phi + y \sin \phi
\]
Image reconstructions from projections

The previous equation can be written as follows:

\[ p_\varphi(u) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \delta(x \cos \varphi + y \sin \varphi - u) \, dx \, dy \]

The Fourier transform of the projection is given by:

\[ P_\varphi(\omega) = \int_{-\infty}^{\infty} p_\varphi(u) e^{-j\omega u} \, du \]

Furthermore, the two-dimensional Fourier transform of \( f(x, y) \) is defined as:

\[ F(\omega_x, \omega_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{-j(\omega_x x + \omega_y y)} \, dx \, dy \]
Image reconstructions from projections

As a special case, we can observe \( F(x, y) \) for \( y = 0 \)

\[
F(x,0) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{-j\omega x} dxdy
= \int_{-\infty}^{\infty} p_0(x) e^{-j\omega x} dx = P_0(\omega)
\]

To conclude: the Fourier transform of a two-dimensional object along the axis \( y = 0 \) is equal to the Fourier transform along the projection angle \( \varphi = 0 \)

In the rotated coordinate system, we have:

\[
\begin{pmatrix}
  u \\
  l
\end{pmatrix}
= \begin{pmatrix}
  \cos \varphi & \sin \varphi \\
  -\sin \varphi & \cos \varphi
\end{pmatrix}
\begin{pmatrix}
  x \\
  y
\end{pmatrix}
\]

\[
P_\varphi(\omega) = \int_{-\infty}^{\infty} p_\varphi(u) e^{-j\omega u} du = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(u,l) e^{-j\omega u} du dl
= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{-j\omega (\cos \varphi x + \sin \varphi y)} dxdy = F(\omega, \varphi),
\]

where

\[
F(\omega, \varphi) = F(\omega_x, \omega_y) \bigg| \begin{array}{c}
\omega_x = \omega \cos \varphi \\
\omega_y = \omega \sin \varphi
\end{array}
\]
Using central slice theorem to solve the CT inverse problem

Align each vector into the 2D spectrum domain using the same angle used for the projection.

2D spectrum of image whose projections are slices which fills the matrix data.

2D spatial inverse Fourier transform (2D IFFT)
Image reconstructions from projections

- Therefore, we can conclude:
  - If we have the object projections, then we can determine their Fourier transforms.
  - The Fourier transform of a projection represents the transform coefficients along the projection line of the object.
  - By varying the projection angle from 0° to 180° we obtain the Fourier transform along all the lines (e.g., we get the Fourier transform of the entire object), but in the polar coordinate system.
  - To use the well-known FFT algorithm, we have to switch from polar to rectangular coordinate system. Then, the image of the object is obtained by calculating the inverse Fourier transform.
  - The transformation from the polar to the rectangular coordinate system can be done by using the nearest neighbor principle, or by using some other more accurate algorithms that are based on the interpolations.
Compressive sensing
Compressive sensing

Compressive sensing approaches have been intensively developed to overcome the limits of traditional sampling theory by applying a concept of compression during the sensing procedure.

Compressive sensing aims to provide the possibility to acquire much smaller amount of data, but still achieving the same quality (or almost the same) of the final representation as if the physical phenomenon is sensed according to the conventional sampling theory.

In that sense, significant efforts have been done toward the development of methods that would allow to sample data in the compressed form using much lower number of samples.
Compressive sensing

- Compressive sensing opens the possibility to simplify very expensive devices and apparatus for data recording, imaging, sensing (for instance MRI scanners, PET scanners for computed tomography, high resolution cameras, etc.).

- Furthermore, the data acquisition time can be significantly reduced, and in some applications even to almost 10 or 20% of the current needs.
WE WILL NEED 6 TIMES LESS MEASUREMENTS FOR THE SAME IMAGE QUALITY

IT MEANS 6 TIMES LOWER EXPOSURE OF PATIENTS TO INVASIVE METHODS

Compressive Sampling in MRI

MRI angiography in lower leg (high-res sampling)

“k-space data”

undersubsampling in FFT domain (undersampled by a factor of 10)

Reconstruction from undersampled data
Standard (left) CS (right)
Compressive sensing

- If the samples acquisition process is linear, than the problem of data reconstruction from acquired measurements can be done by solving a linear system of equations.
- Measurement process can be modelled by the measurement matrix $\Phi$.
- Signal $f$ with $N$ samples can be represented as a signal reconstruction problem using a set of $M$ measurements obtained by using $\Phi$ as follows:
  $$\Phi f = y$$
- Where $y$ represents the acquired measurements.
CS conditions

- CS relies on the following conditions:
  - **Sparsity** – related to the signal nature;
    - Signal needs to have concise representation when expressed in a proper basis \( \Psi \) \((K<<N)\)
  - **Incoherence** – related to the sensing modality; It should provide a linearly independent measurements (matrix rows)
  - **Random** undersampling is crucial

- **Restriced Isometry Property** – is important for preserving signal isometry by selecting an appropriate transformation
Standard approach for signal sampling and its **compressive sensing** alternative
CS problem formulation

- The method of solving the undetermined system of equations \( y = \Phi \Psi x = Ax \), by searching for the sparsest solution can be described as:

\[
\min \|x\|_0 \quad subject \ to \ y = Ax
\]

- We need to search over all possible sparse vectors \( x \) with \( K \) entries, where the subset of \( K \)-positions of entries are from the set \( \{1, \ldots, N\} \). The total number of possible \( K \)-position subsets is \( \binom{N}{K} \).
A more efficient approach uses the near optimal solution based on the $l_1$-norm, defined as:

$$\min \|x\|_1 \quad subject \ to \ y = Ax$$

- In real applications, we deal with noisy signals.
- Thus, the previous relation should be modified to include the influence of noise:

$$\min \|x\|_1 \quad subject \ to \ \|y - Ax\|_2 \leq \varepsilon$$

$$\|e\|_2 = \varepsilon$$

L2-norm cannot be used because the minimization problem solution in this case is reduced to minimum energy solution, which means that all missing samples are zeros.
Summary of CS problem formulation

Signal $f$ - linear combination of the orthonormal basis vectors

$$f(t) = \sum_{i=1}^{N} x_i \psi_i(t), \text{ or }: \ f = \Psi x.$$ 

Set of random measurements:

$$y = \Phi f$$
CS reconstruction algorithms

- I group: convex optimizations such as
  - Basis pursuit
  - Dantzig selector,

- Greedy algorithms:
  - Matching pursuit,
  - Orthogonal matching pursuit;
  - Iterative thresholding (hard and soft versions)

- Hybrid methods:
  - Compressive sampling matching pursuit
Greedy algorithms – Orthogonal Matching Pursuit (OMP)

\[ \Psi - \text{Transform matrix} \]
\[ \Phi - \text{Measurement matrix} \]
\[ y = \Phi f - \text{Measurement vector} \]
Iterative hard thresholding

- The IHT algorithm is an iterative method

\[ x^{n+1} = H_k(x^n + \mu \Phi^T(y - \Phi x^n)), \]

where \( H_k \) is the hard thresholding operator that sets all but the \( k \) largest (in magnitude) elements in a vector to zero.

- IHT is a simple reconstruction algorithm and it can recover sparse and approximately sparse vectors with near optimal accuracy.

- 1) The step size \( \mu \) has to be chosen appropriately to avoid instability of the method

- 2) IHT has only a linear rate of convergence
Single-Iteration Reconstruction
Threshold based Algorithm for components

$y$ – measurements
$M$ - number of measurements
$N$ – signal length
$T$ - Threshold

- DFT domain is assumed as sparsity domain
- Apply threshold to initial DFT components (determine the frequency support)
- Perform reconstruction using identified support

\[ T = \sqrt{-\sigma^2 \log(1-P(T)^{\frac{1}{N}})} \]

\[ X = DFT\{y\} \]

\[ k = \arg\left\{ \{X| > \frac{T}{N}\} \right\} \]

\[ X_R = (Acs^*Acs)^{-1}Acs^*y \]

START

\[ y, N, M \]

END
Variance of DFT values at the non-signal and signal positions can be calculated as:

\[ \sigma^2 = \text{var}\{F_{k \neq k_i}\} = \frac{M(N - M)}{(N - 1)} \sum_{k=1}^{K} A_i \]

\[ y = \Phi \Psi x = \Theta x \]

\( \Theta \) is a CS matrix (could be DFT, DCT, HT)
In each iteration we need to remove the influence of previously detected components and to update the value of threshold.
Compressive sensing in biomedical imaging
Compressed sensing in CT

- Due to its powerful ability of reconstructing signal or image from highly undersampled data, CS has attracted tremendous attention in medical imaging community.
- The first attempt to apply CS to medical imaging was done by Lustig et al., in which they successfully reconstructed MR images with 40% of the data.
- There are also research efforts on compressed sensing CT because in principle a CS based algorithm can produce good reconstructions using fewer samples (projections), it has the potential to reduce the radiation exposure to the patient.
Recovering of highly undersampled signal

512*512 Shepp-Logan

Undersampled by 22 radial lines

Normal Reconstruction

CS reconstruction algorithm
Compressed sensing in CT

- The most popular type of method that has been studied is the Total Variation (TV) based compressed sensing methods.
- TV based methods use total variation as a sparsity transform.
Total Variation minimization

- Natural images are not sparse – neither in space nor in the frequency domain.
- *Instead of the* $l_1$ *norm minimization, Total Variation (TV) minimization is commonly used.*
- Having in mind that the image gradient is sparse, TV is, in fact, $l_1$ norm minimization of the image gradient.
- Introduced by Rudin, Osher and Fatemi, for solving the inverse problems.
Total Variation minimization

- Inverse problems:
  - signal $X$, measurements $y$ (modeled by applying the operator $A$ – measurement matrix)
  - Noisy measurements: $y=AX+n$
  - Linear inversion (if $A$ is linear), deconvolution problem
  - Standard approach to linear inversion problems:
    - **Definition of the objective function**
    - **Solution of the minimization function according to** $X$
Total Variation minimization

\( \mathbf{X} \) - signal to be estimated
\( \mathbf{n} \) – additive noise
\( \mathbf{A} \) - matrix that models measurement process

Defining the objective function:

\[ F(\mathbf{X}) = d(\mathbf{y}, \mathbf{AX}) + \lambda R(\mathbf{X}) \]

- difference measure between signals \( \mathbf{y} \) and \( \mathbf{X} \)
- it can be mean square error \( d(\mathbf{y}, \mathbf{AX}) = \| \mathbf{y} - \mathbf{AX} \|_2^2 \)

\( R(\mathbf{X}) \) – regularization function
\( \lambda \) – regularization parameter, \( \lambda > 0 \)

The value of the \( \mathbf{X} \) should be such that the relation \( \mathbf{AX} \) corresponds to the vector \( \mathbf{y} \)
Total Variation minimization

- Solution $X = A^{-1}y$ is not applicable if $A$ is not an invertible matrix.
- The goal of $R(X)$ is to avoid such situations.
- If the signal $X$ is sparse, the function $R(X)$ corresponds to the $l_1$ norm, i.e. objective function is defined as:

$$F(X) = \|y - AX\|_2^2 + \lambda \|X\|_1$$
Total Variation minimization

- The regularization function $R(X)$ can be defined as TV norm:

$$ R(X) = \|\nabla X\|_1 $$

- The $\nabla$ is the gradient operator:

$$ \nabla_{i,j} X = \begin{bmatrix} X(i+1,j) - X(i,j) \\ X(i,j+1) - X(i,j) \end{bmatrix} $$

- TV norm in the discrete form:

$$ TV(X) = \sum_{i,j} \sqrt{(\nabla^h_X)^2 + (\nabla^v_X)^2} $$

$$ \begin{align*}
\nabla^h_X &= X(i+1,j) - X(i,j) \\
\nabla^v_X &= X(i,j+1) - X(i,j)
\end{align*} $$

- Row and column differences
Iterative shrinkage/thresholding algorithms

- Special attention is paid to the **Iterative shrinkage/thresholding (IST)** algorithms.
- Used for solving multi-dimensional optimization problems.
- Problem of these algorithms: **slow convergence**
- Convergence of the IST algorithms is speed up by introducing the two step IST algorithm (**two-step Iterative shrinkage/thresholding, TwIST**).
Two-step Iterative shrinkage/thresholding, TwIST

- Finds solution $X$ in two-steps:
  
  $$
  X_1 = \Gamma_{\lambda}(X_0),
  
  X_{t+1} = (1 - \alpha)X_{t-1} + (\alpha - \beta)X_t + \beta\Gamma_{\lambda}(X_t), \ t \geq 1.
  $$

- Where $X$ is defined as:
  
  $$
  \Gamma_{\lambda}(X) = \Psi_{\lambda}(X + A^T(y - AX))
  $$

- Denoising and regularization functions are described by following relations:

  $$
  \Psi_{\lambda}(\varepsilon) = \arg\min_{X} \frac{1}{\mu} \Phi_{\text{reg}}(X) + \frac{1}{2}\|X - \varepsilon\|^2
  $$

  Denoising operator

  $$
  \Phi_{\text{reg}}(X) = \sum_i \|D_iX\|
  $$

  Regularization function for TV l1 problems
Two-step Iterative shrinkage/thresholding, TwIST

- Optimal choice for the parameters $\alpha$ and $\beta$:

\[
\alpha = \left(\frac{1-\sqrt{k}}{1+\sqrt{k}}\right)^2 + 1, \quad \beta = \frac{2\alpha}{\max(\zeta_m) + \zeta_1}
\]

- There is a large number of methods for solving TV minimization problems: *time marching scheme*, *fixed point iteration method*, *majorization-minimization approach*

- TV-L2 problem can be defined as:

\[
\min_{X} \sum_{i} \sqrt{\|D_iX\|^2 + \varepsilon} + \frac{\mu}{2} \|AX - y\|^2
\]

\[0 < \zeta_1 \leq \lambda_i(A^T A) \leq \zeta_m,\]

\[k = \frac{\zeta_1}{\max(\zeta_m)}\]

$\varepsilon$ - constant

$y$ measurements

$X$ – image to be obtained

$A$-CS matrix
Image reconstruction examples

Original image

Reconstructed images

DFT masks
Image reconstruction – more examples

FFT domain mask

DCT domain mask

Reconstructed image

Reconstructed image

Reconstructed image

Reconstructed image
ECG signals
ECG signals - basics

- ECG signal represents the activity of the heart.
- The propagation of electrical waves resulted from cardiac activity can be measured as a difference of potentials between two points on the human body.
- In the case of multi-channel ECG, the signals are measured between different pairs of electrodes on the human body.
- ECG signal characteristics can be described using 7 specific parts.
  - P,Q,R,S and T peaks
  - 2 iso-electric parts that separate P and T peak from the large QRS complex.
ECG signals - basics

![ECG Signal Diagram]

- PP interval
- RR interval
- Voltage [mV]
- P complex
- PQ interval
- QRS complex
- ST segment
- QT interval
- Idealized cardiac cycle
- Time [sec]
ECG-signal basics

- The heart rhythm is of sinus type characterized among other features by the presence of P-complex before every QRS complex.
- The heart impulse rate is characterized by heart agitation in average in the frequency of 60–100 beats per minute (bpm) for adults, although it might be normal, especially for athletes, to have resting bpm as low as 30.
- Deviations of the rate beyond this range can be treated as abnormal case. However, the limit frequencies should be taken individually.
- For the time duration of PR, QRS and QT parts in the cardiac cycle, a reasonable rule is to consider that the interval QT is less than a half of the distance between two successive QRT complexes. That is, QT should be less than 1/2 of the RR interval.
- P complex (P wave) is normally 0.04–0.11 s in duration. Its deviation from the normal wave shape or its disappearing means a pathological case.
- The normal duration of ST segment is 0.02–0.12 s. Any drop in the duration of the ST segment suggests ischemic, whilst its shift above the cycle-axis suggests a heart attack.
- The normal T-complex is about half of the P-complex time. When it is inverted (except for aVR lead), it typically indicates a heart attack.
Recorded evolution of ECG waveform showing the pathological disturbances after the heart attack in ST, T and Q parts.

<table>
<thead>
<tr>
<th>Cardiac cycle</th>
<th>Minutes after heart attack starts</th>
<th>Hours later</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>ST shifted</td>
<td>- ST shifted</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- R dropping</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- Q appears to be noticeable</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1-2 days</th>
<th>Several days</th>
<th>Weeks after the attack</th>
</tr>
</thead>
<tbody>
<tr>
<td>- T inverted</td>
<td>- ST normal</td>
<td>- ST and T normal</td>
</tr>
<tr>
<td>- Q goes lower</td>
<td>- T inverted</td>
<td>- Q no change</td>
</tr>
</tbody>
</table>
Finding of proper heartbeats on the example of R and P points. The signal and the markings of characteristic points are drawn by the simulation software.
The signal split into 0.25-mV high and 30-second wide spans.

Inside these spans all minima and maxima are counted. Based on their numbers, the span of the signal is estimated, and the mV spans containing possible R and S points are acquired.

On the basis of these spans, the probable P and T points are found.

The last part is the search for Q point and ST segment with the use of least squares approach.

A training algorithm can be used to obtain the ECG signal pattern.

The algorithm is based on statistical analysis of probabilities of the existence of characteristic points of ECG signal in the given millivolt (mV) and time frames.
ECG signals and QRS complexes

- ECG signals represent the records of electrical activity of the heart over a period of time.
- QRS complexes are the most characteristic waves in ECG signals.
- QRS complexes are crucial in different stages of medical diagnosis and treatment of heart deceases.
- Important research efforts have been made in the development of recognition, classification and compression algorithms for QRS complexes.
- Effective storage, automatic detection of anomalies and automatic diagnosis based on the processing of QRS complexes are the state-of-the-art aims in modern biomedicine.
QRS complexes in ECG signals

First 10 milliseconds of a real ECG signal
Red parts denote intervals of QRS complexes

PhysioNet: MIT-BIH ECG Compression Test Database
http://www.physionet.org/physiobank/database/cdb
QRS complexes and the Hermite transform

- The \( p \)-th order Hermite function and the functions of the \( p-1 \) and \( p-2 \) orders and can be related with the following recursive formula:

\[
\psi_0(t, \sigma) = \frac{1}{\sqrt{\sigma \sqrt{\pi}}} e^{-\frac{t^2}{2\sigma^2}}, \quad \psi_1(t, \sigma) = \sqrt{\frac{2}{\sigma \sqrt{\pi}}} e^{-\frac{t^2}{2\sigma^2}},
\]

\[
\psi_p(t, \sigma) = \frac{t}{\sigma} \sqrt{\frac{2}{p}} \psi_{p-1}(t, \sigma) - \sqrt{\frac{p-1}{p}} \psi_{p-2}(t, \sigma).
\]

- Hermite transform is defined by:

\[
f(t) = \sum_{p=0}^{M} c_p \psi_p(t, \sigma), \text{ while the Hermite coefficients are:}
\]

\[
c_p = \frac{1}{M} \sum_{m=1}^{M} \frac{\psi_p(t_m, \sigma)}{[\psi_{M-1}(t_m, \sigma)]^2} f(t_m), \quad p = 0, 1, \ldots, M - 1
\]
First four Hermite basis functions

Only few Hermite coefficients are needed for the representation of QRS complexes

- Visual similarity between QRS complexes as signals with compact time support and Hermite bass functions is obvious
- Thus, Hermite transform is often incorporated in classification and compression algorithms for QRS complexes.
- A QRS complex can be modeled as:

\[ s(t) = \sum_{p=0}^{K} c_p \psi_p (t, \sigma) \]

- Here, \( K \) is the number of non-zero Hermite coefficients needed for the successful representation of QRS complex
QRS complexes and the Hermite transform: Representation and the CS scenario
The influence of the Hermite scaling factor

- Hermite transform of the QRS complex:
- first row – the selected QRS complex,
- second row – Hermite coefficients of the signal with $\sigma = 1 \cdot \Delta t$,
- third row – Hermite coefficients of the signal with $\sigma = 5.7 \cdot \Delta t$.

*Automatic procedure of scaling factor optimization?*
A compression procedure

- Use a suitably chosen scaling factor
- Approximate the signal with $K$ most significant HT coefficients, such that the relative error:

$$E = \frac{\|\tilde{f} - f\|_2}{\|f\|_2} < 10\%$$

where $\tilde{f}$ is the inverse HT of the $K$ most significant coefficients, and $f$ is the vector of the original signal.

- Our experiments show that only 4-5 Hermite coefficients are needed to be stored in order to preserve a medically acceptable error level.

- Very important in order to generate databases for every patient with heart disease, in order to make efficient diagnosis and anticipation of future problems.
A compression procedure

- Original and approximated QRS complex
- Only 4 HT coefficients are used, signal optimally resampled

![Diagram](https://via.placeholder.com/150)

Amplitude [mV]

- Original QRS complex
- Hermite transform of the original QRS complex
- Signal resampled by using optimal $\lambda$ and $m$
- Hermite transform of the resampled signal
Challenges

- Automatic optimization of the scaling factor using concentration measures
- Apply concentration measures in order to determine a threshold for the automatic recognition and/or classification of QRS complexes in ECG signals
- Use the Hermite transform as the core in expert systems and classification algorithms
- Results verification in consultation with cardiologists
Detection of swallowing sounds – appl. Dysphagia
Scope of the approach

- Innovative approach for the analysis of one-dimensional biomedical signals that combines the Hermite projection method with time-frequency analysis.
- A two-step approach to characterize vibrations of various origins in swallowing accelerometry signals.
  - First, by using time-frequency analysis we obtain the energy distribution of signal frequency content in time.
  - Second, by using fast Hermite projections we characterize whether the analyzed time-frequency regions are associated with swallowing or some other phenomena (vocalization, noise, bursts, etc.).
- The numerical analysis of the proposed scheme clearly shows that by using a few Hermite functions, vibrations of various origins are distinguishable.
1. **Swallowing Difficulties**

- Deglutition, or swallowing, is a well-defined, complex process of transporting food or liquid from the mouth to the stomach.
- Swallowing consists of four phases: oral preparatory, oral, pharyngeal, and esophageal.
- Patients suffering from dysphagia (swallowing difficulty), usually deviate from the well-defined pattern of healthy swallowing.
- Dysphagia is a common problem encountered in the rehabilitation of stroke patients, head injured patients, and others with paralyzing neurological diseases.
- Today's dysphagia management relies heavily on the videofluoroscopic swallowing study (VFSS).
- VFSS is accepted as the gold standard, but requires expensive X-ray equipment and expertise from speech-language pathologists and radiologists.
2. Cervical auscultation and Swallowing accelerometry

Cervical auscultation is a promising non-invasive tool for the assessment of swallowing disorders, adopted by dysphagia clinicians.

Cervical auscultation approach involves the examination of swallowing signals acquired via a stethoscope or other acoustic and/or vibration sensors during deglutition.

Swallowing accelerometry, a technique that involves an accelerometer placed on the neck to monitor vibrations associated with swallowing activities, has been used to detect aspiration in several studies.

Nevertheless, the presence of various vibrations not associated with swallowing can severely contaminate swallowing accelerometry signals.

Vocalizations either voluntarily or involuntarily can have an adverse effect on these signals, whose presence masks the observed swallowing signals.
3. Data

The sample data considered in this paper were
gathered over a three month period from a public science centre in Toronto, Ontario, Canada. A dual-axis accelerometer (ADXL322, Analog Devices) was attached to the participant’s neck using double-sided tape in order to monitor vibrations associated with swallowing.

Data were band-pass filtered in hardware with a pass band of 0.1-3000 Hz and sampled at 10kHz using a custom LabVIEW program running on a laptop computer. Data were saved for subsequent off-line analysis.
During data collection, participants were cued to perform three types of swallows involving saliva and water swallows. The entire data collection session lasted 15 minutes per participant. The participants were instructed not to vocalize. Nonetheless, approximately one quarter of all recordings contained either voluntary or involuntary vocalizations.
Time-Frequency Analysis

- Signals considered in this paper mostly contain swallowing vibrations.
- Some recordings also contain vibrations associated with vocalization (e.g., speech, cough, laughter) and various burst components produced by the equipment and noise.
- The most dominant vibrations are those produced by swallowing and vocalization.
- Fortunately, vibrations associated with different phenomena provide unique time-frequency signatures.
Thus, time-frequency representations are crucial for the analysis and classification of these signals. Also, having in mind multicomponent nature of these signals, time-frequency representations without cross-terms should be used.

Time-frequency regions of spectrogram for voiced sound (left), for swallowing sound (right)
Hermite projection method applied to time-frequency regions

To understand differences between swallowing vibrations and other miscellaneous vibrations (speech, laugh, cough, etc.) we observe the structures in the time-frequency regions. Here, the spectrogram, as the simplest time-frequency distribution, is used:

A certain pre-processing in classification procedure is applied, since the signal could be corrupted by noise: the time-frequency mask is defined to remove the noise influence and locate the significant signal components:

\[
L(t, \omega) = \begin{cases} 
1 & \text{for } SPEC(t, \omega) \geq \xi \\
0 & \text{otherwise} 
\end{cases}
\]

where the threshold value \(\xi\), i.e., the energy floor is obtained as:

\[
\xi = 10^{\sigma \log_{10}(\max_{t, \omega}(SPEC(t, \omega)))}
\]
Hermite projection method applied to time-frequency regions

- Filtered spectrogram is given by:
  
  \[ \text{SPEC}_{\text{filt}}(t, \omega) = L(t, \omega) \text{SPEC}(t, \omega) \]

- The energy vector is calculated to determine the time intervals containing vibration activities:
  
  \[ E(t) = \sum_{\omega} \left| \text{SPEC}_{\text{filt}}(t, \omega) \right| \]

- Consequently, the time support vector is obtained as (threshold \( \beta \) is used to remove the remaining noise):
  
  \[ E_f(t) = \begin{cases} 
  1 & \text{for } E(t) \geq \beta \\
  0 & \text{otherwise}
  \end{cases} \]

- The time-frequency regions for classification are extracted from \( \text{SPEC}_{\text{filt}}(t, \omega) \) for \( t = \arg\{ E_f(t) = 1 \} \).

- The fast Hermite projection method is applied to these regions.
Filtered time-frequency representation and assigned regions (upper row), time support vector (bottom row)
Hermite projection method applied to time-frequency regions

- We use a small number of functions for reconstruction such that:
  - simple structure regions are reconstructed with a small error
  - complex structure regions with a significantly larger error.

- The difference between the original and reconstructed regions i.e. the error, which depends on the region's structure of a region, will be used for characterization. This difference is measured by the mean squared error, as follows:

\[
MSE = \frac{1}{T \Omega} \sum_t \sum_\omega (R(t, \omega) - R'(t, \omega))^2
\]

- where \( R(t, \omega) \) denotes the original region, \( R'(t, \omega) \) is the reconstructed region.
It has been shown that the voiced vocalization regions (speech, cough, laughter, etc.) are more complex, and thus have the higher MSE, in comparison to the regions with containing swallowing sounds vibrations.

Moreover, there is a significant gap between the values of MSE for voiced vocalization and for swallowing sounds vibrations. On the other hand, the MSE for regions with swallowing sounds vibrations is much higher than for the regions with containing noise.

The difference between the original and reconstructed signals is quantified for each region in terms of MSE. This parameter is used to characterize a region based on the table:

<table>
<thead>
<tr>
<th>No.</th>
<th>Region description</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Noise</td>
<td>0.086</td>
</tr>
<tr>
<td>2</td>
<td>Noise</td>
<td>0.01</td>
</tr>
<tr>
<td>3</td>
<td>Noise</td>
<td>0.22</td>
</tr>
<tr>
<td>4</td>
<td>Swallowing vibrations</td>
<td>36.29</td>
</tr>
<tr>
<td>5</td>
<td>Burst</td>
<td>0.13</td>
</tr>
<tr>
<td>6</td>
<td>Burst</td>
<td>3.4</td>
</tr>
<tr>
<td>7</td>
<td>Vocalization</td>
<td>669</td>
</tr>
<tr>
<td>8</td>
<td>Vocalization</td>
<td>1057.2</td>
</tr>
<tr>
<td>9</td>
<td>Vocalization</td>
<td>762</td>
</tr>
<tr>
<td>10</td>
<td>Vocalization</td>
<td>505</td>
</tr>
<tr>
<td>11</td>
<td>Noise</td>
<td>0.88</td>
</tr>
<tr>
<td>12</td>
<td>Swallowing vibrations</td>
<td>18.6</td>
</tr>
<tr>
<td>13</td>
<td>Noise</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>1.</td>
<td>2.</td>
</tr>
<tr>
<td>---</td>
<td>----</td>
<td>----</td>
</tr>
<tr>
<td>Vocalizations</td>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
</tr>
<tr>
<td>MSE</td>
<td>1999</td>
<td>988</td>
</tr>
<tr>
<td></td>
<td>7.</td>
<td>8.</td>
</tr>
<tr>
<td>Vocalizations</td>
<td><img src="image7.png" alt="Image" /></td>
<td><img src="image8.png" alt="Image" /></td>
</tr>
<tr>
<td>MSE</td>
<td>1439,1</td>
<td>3093,8</td>
</tr>
<tr>
<td>Swallowing Vibrations</td>
<td><img src="image13.png" alt="Image" /></td>
<td><img src="image14.png" alt="Image" /></td>
</tr>
<tr>
<td>MSE</td>
<td>20</td>
<td>16</td>
</tr>
</tbody>
</table>

**The illustrations of several zoomed time-frequency regions and corresponding MSEs**

Telemedicine

- Humans tend to live longer
- Increased pressure on healthcare systems worldwide to provide higher quality health care
- Telemedicine promotes the use of multimedia services and systems for increasing the availability of care for patients
- Telemedicine provides a way for patients to be examined and treated, while the health care provider and the patient are at different physical locations.
- Telemedicine technologies - services to patients using signals that can be acquired over distances.
- Signal and image transmission, storage and processing are the major components of telemedicine.
Telemedicine

- Telenurcing
- Telepharmacy
- Telerehabilitation
- Teleradiology
- Telecardiology
- Telesurgery
Telenursing

- Provide home care to older adults and/or other patient groups, which preferred to stay in the comfort of their own homes.
- Generally, the patients welcome the use of multimedia systems to communicate with a nurse about their physical and psychological conditions.
- An analysis of telenursing in the case of metered dose inhalers in a geriatric population have shown that multimedia systems can provide most of services, and only small percentage require on-site visits.
Telepharmacy

- Assumes providing pharmaceutical care to patients and medication dispensing from distance.
- The telepharmacy services adhere to all official regulations and services as traditional pharmacies, including verification of drugs before dispensing and patient counseling.
- Telepharmacy services maintain the same services as the traditional pharmacies and provide additional value-added features.
- Specifically, it has been shown that the utility of telepharmacy services for education via video was superior to education provided via written instructions on an package insert.
Telerehabilitation

Telerehabilitation was established in 1997 when the National Institute on Disability and Rehabilitation Research (U.S. Department of Education) ranked it as one of the top priorities for a newly established Rehabilitation Engineering Research Center (RERC). It covers diverse fields of investigations (e.g., intelligent therapeutic robots and other health gadgets).

The main efforts are made to:

- provide telecommunication techniques to support rehabilitation services at a distance,
- then to provide technology for monitoring and evaluating the rehabilitation progress,
- and finally to provide technology for therapeutic intervention at a distance
Telerehabilitation

A typical telerehabilitation system
Telecardiology

- Telecardiology involves merging technology with cardiology in order to provide a patient with a proper medical care.

- Telecardiology is currently a well-developed medical discipline involving many different aspects of cardiology:
  - acute coronary syndromes,
  - congestive heart failure,
  - sudden cardiac arrest, arrhythmias).

Telecardiology

- It becomes an essential tool for cardiologists.
- Patient consultations with cardiologists via multimedia systems are becoming extremely common: a cardiologist receives many signals and images in real time to assess the patient condition.
- Telecardiology has the two major aims:
  - to reduce the healthcare cost.
  - to evaluate the efficiency of telecardiac tools (e.g., wireless ECG) at variable distances.

By accomplishing these two aims, telecardiology will enhance the psychological well-being of patients in addition to bridging the gap between rural areas and hospitals.
Telesurgery

- Dissemination of new surgical skills and techniques across the wide spectrum of practicing surgeons is often difficult and time consuming, especially because the practicing surgeons can be located very far from large teaching centers.
Telesurgery

- It allows:
  - dissemination of expertise,
  - widespread patient care,
  - cost savings, and
  - improved community care
- Telesurgery seems to be a powerful method for performing minimally invasive surgery
- Operators using a telesurgery platform can complete maneuvers with delays up to 500 ms.
- The emulated surgery in animals can be effectively executed using either ground or satellite, while keeping the satellite bandwidth above 5 Mb/s.
Literature

Literature


- M. Brajovic, I. Orovic, M. Dakovic, S. Stankovic, "Gradient-based signal reconstruction algorithm in the Hermite transform domain," *Electronic Letters*, Accepted for publication, 2015

Literature

Literature